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COMMENT

Comment on Németh's arguments on the phase diagram of spin glasses

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Abstract. Imperfections are pointed out in the arguments of Németh on the possibilities of straight phase boundaries in the phase diagram of Ising spin glasses.

Németh (1987) recently published arguments on the possible structure of phase diagrams of the Ising spin glasses. His claims are: (i) that the boundary between the paramagnetic and spin-glass (SG) phases is straight (horizontal) in a finite neighbourhood of the symmetric bond distribution (e.g. $p=\frac{1}{2}$ in the case of the $\pm J$ model), and (ii) that the boundary between the ferromagnetic and SG phases is straight (vertical). In the present comment I point out imperfections in his arguments in drawing these conclusions.

In his derivation of (i) he expands the partition function $Z(\beta)$ in powers of $\tanh \beta J$ assuming analyticity of $Z(\beta)$ in a finite range of the inverse effective temperature β. However, it is meaningless to talk about analyticity of the partition function if the system size N is infinite. The free energy per site f may be an analytic function, but $Z(\beta)=\exp(-\beta f N)$ is either divergent or vanishing. If N is finite, then his expansion (8) is trivial because the LHS (the SG order parameter) identically vanishes. One should apply an appropriate symmetry breaking field to have a finite SG order parameter, but then equation (4) fails to hold. It is legitimate to replace a ferromagnetic symmetry breaking field with the condition of all-up spins at the boundary sites (Nishimori 1981), in which case his equation (1) is correct. However, this replacement loses its sense in the case of the SG order parameter (4).

In his argument for point (ii), replacement of equation (9) by (10) is invalid because the sign of a single-site average $\langle S_i \rangle$ may change at many sites in lowering the temperature from β (in the ferromagnetic phase) to β_0 (in the SG phase) if there is a re-entrance for a typical bond configuration. The reason is that otherwise the average of $\langle S_i \rangle$ cannot vanish at β_0 (SG phase) when it is finite at β (ferromagnetic phase). Thus in proceeding from (9) to (10) he implicitly assumes the absence of re-entrance. Even if one accepts his inequality (12), the problem of a symmetry breaking field (and/or the boundary condition) as mentioned above hampers his interpretation that each factor in the RHS of (12) represents the SG order parameter.

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Let us finally note that his result (ii) has already been derived from more physically plausible arguments (Nishimori 1986).

References

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